Table 1 1% peak resultant bending moments at various sites as a ratio of the 1% peak resultant bending moments calculated for Montgomery, Ala.—vehicles A, B, and C

	Vehicles		
Site	A	B	C
Long Beach	0.950	0.766	1.014
Denver	0.952	1.159	0.791
Seattle	0.875	0.935	0.957
Fort Worth	0.956	0.970	0.923
International Falls	0.905	1.033	0.818
Caribou	1.213	1.413	1.074
Kadena	1.132	1.182	1.080
Tripoli	0.994	1.145	0.976
Bitburg	0.838	0.946	0.962
Kehavik	1.122	1.167	1.060

by a certain vehicle flying a sample of 200 Montgomery, Ala. wind soundings. This distribution of peak loads deviates substantially from a normal distribution, in which all points would plot in a straight-line path. As a matter of fact, for some vehicle-site combinations, the 1% load obtained from a normal distribution with the same mean and same standard deviation would be as much as 20% lower than that obtained from the likes of Fig. 1.

For the vehicles and GMD-1 wind samples considered here, the calculation of 1% loads for each sample of 200 soundings varied between 2.5 and 5.5 hr of IBM 7090 time, depending upon the time increment needed to insure a stable numerical solution. The load calculation procedure is applicable to wind soundings of any degree of coarseness; the finer the wind sounding, however, the more is the machine time required to compute the loads.

Discussion of Results

Table 1 presents the 1% peak bending moments for vehicles A, B, and C as a ratio of the 1% peak bending moments calculated at the Montgomery, Ala. site. The most severe loads on vehicles A and B are obtained for the Caribou wind sample, whereas for vehicle C, the Kadena site is most critical.

The differences between the 1% loads for Caribou and Montgomery for vehicles A and B, 21% and 41% greater, respectively, at Caribou, are particularly surprising. Climatological data in Ref. 4 indicate that, at the altitude of maximum winds, the Montgomery winds are more severe than the Caribou winds. However, the results in Table 1 are partially explanable upon close study of the wind samples. Of all the samples considered, the Caribou sample is the only one for which the altitude of the highest 1% wind differs from the altitude of the highest mean wind. This 1% wind for Caribou is higher than that of any other site even though the mean winds at Caribou are lower than some others. The Caribou sample also shows more very high shears than any of the other sites and more variation in wind direction with altitude.

The Keflavik wind sample possesses many of the same characteristics as the Caribou sample and, therefore, also provides higher loads than those from Montgomery. It was expected that the Kadena and Tripoli winds might provide more severe loads than the winds from Montgomery because of the higher winds reported at these locations. This expectation is verified in all cases for Kadena and for vehicle B at Tripoli.

Some interesting variations in loading are noticeable among all the United States sites other than Caribou. At the Long Beach site, the 1% loads for vehicles A and C are very close to those for the Montgomery site, whereas on vehicle B they are 24% lower than the Montgomery loads. It also is worth noting that Denver proved to be the second most severe United States site for vehicle B while being the least severe for vehicle C. The least severe United States sites for vehicles A, B, and C are Seattle, Long Beach, and Denver, respectively.

Although Caribou winds provided the most severe loads of any of the United States sites considered, the severity of loading for other sites varies widely among the different vehicles studied. This behavior suggests that wind characteristics alone do not determine the load response of a vehicle and that vehicle parameters—such as aerodynamic characteristics, thrust-to-weight ratios, and control system characteristics—also may be quite important. It suggests, furthermore, that Caribou may not be the most critical United States site for new vehicles.

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Effects of Surface Curvature on Laminar Boundary-Layer Flow

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SOME authors have investigated the effects of surface longitudinal curvature K on laminar boundary-layer flow for the case where the potential flow velocity U_1 is constant and $K \propto x^{-1/2}$, x being the distance along the surface. Tani[1] developed a small perturbation theory neglecting terms of $0(A^2)$, where $A = K(\nu x/U_1)^{1/2}$, and showed that the skin friction coefficient C_f increases with decrease of A. Murphy[2] independently analyzed the same problem by similarity consideration and obtained similar results. Recently, Yen[3, 5] and Toba[4, 5] re-examined the problem and found inverse effects. The purpose of this note is to clarify the reason for this discrepancy and to obtain the correct effects of surface curvature on laminar boundary-layer flow.

First it will be shown that Murphy's Eq. (32) and Yen's Eq. (A-3) are essentially the same. Yen's Eq. (A-3) can be integrated to give

$$f''' + (f + 2C)f'' = Be^{-2C\chi} \quad (1)$$

provided $\beta = 0$, where B is an integration constant and primes denote differentiation with respect to χ. Note that their numerical work is concerned with this value of β. Now it can be shown that B must be zero in order that f'' should tend to zero as $\exp(-\chi^2/2)$ when $\chi \rightarrow \infty$. With the transformation $\phi = f + 2C$, Eq. (1) reduces to

$$\phi''' + \phi\phi'' = 0 \quad (2)$$

The boundary conditions are $\phi(0) = 2C$, $\phi'(0) = 0$, $\phi'(\infty) =$

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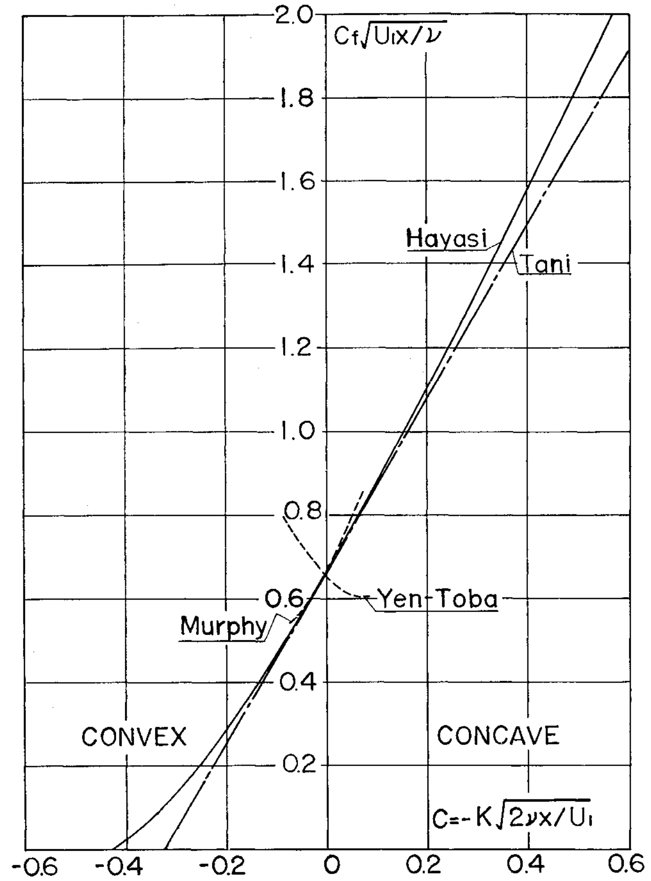

Fig. 1 Skin friction coefficient as a function of surface curvature

1. From Eqs. (38) and (52) of Yen and Toba's paper, one has

$$n = -K^{-1}(1 - e^{-C\chi}) \tag{3}$$

where n is the distance along a curvilinear coordinate line. Therefore one obtains, from their Eqs. (46) and (A-1), $v_\xi = U_1f'/(1 + Kn)$, where v_ξ is the velocity. In potential flow, one gets from their Eq. (60)

$$u_{\text{pot}} = U_1/(1 + Ky) \tag{4}$$

where y is the distance normal to the surface. Since $v_\xi \doteq u$ and $n \doteq y$, one has

$$f' = \phi' = u/u_{\text{pot}} \tag{5}$$

Therefore ϕ' is the ratio of the velocity in the boundary-layer flow to that in the potential flow on the same normal. Expressing Murphy's f as F, one has, from Eqs. (4) and (5) and Murphy's Eq. (30), $dF/d\eta = 2f'/(1 + 2A\eta)$. Since $A \doteq -2^{-1/2}C$, one obtains from Eq. (3)

$$\chi = 2^{-1/2}A^{-1}\ln(1 + 2A\eta) \tag{6}$$

Thus one has $F = 2^{1/2}f$. Therefore Murphy's Eq. (32) reduces to the present Eq. (2), which was derived from Yen and Toba's equation. Equation (6) shows that $\chi = 0$ at $\eta = 0$ and that $\chi \to \infty$ when $\eta \to \infty$ for $A \geq 0$, or when $\eta \to -(2A)^{-1}$ for $A < 0$. Therefore, the boundary conditions for F also reduce to those for ϕ. Thus it is clear that Murphy's and Yen's analyses are essentially the same, and both are correct.

For the purpose of numerical calculation, Eq. (2) is very convenient. Indeed, with the transformations $\phi = ky$, $\chi = k^{-1}X$, one has $y''' + yy'' = 0$, together with boundary conditions $y(0) = 2Ck^{-1}$, $y'(0) = 0$, $y'(\infty) = k^{-2}$. Thus, if a solution y', satisfying the initial conditions $y(0) = \alpha$, $y'(0) = 0$, $y''(0) = \beta$, tends to γ when $X \to \infty$, then one can obtain a solution of Eq. (2) from the initial conditions $\phi(0) = 2C = \alpha\gamma^{-1/2}$, $\phi'(0) = 0$, and $\phi''(0) = \beta\gamma^{-3/2}$. In such a way, numerical solutions of Eq. (2) have been obtained on an electronic digital computer Datatron 205. The integration was done by using the Runge-Kutta method with fourth-order accuracy. The interval of calculation is 0.01. The variation of C_f is plotted in Fig. 1, where the results of Murphy and Yen and Toba and the line representing Tani's formula[1]

$$C_f = 0.664 + 2.05C \tag{7}$$

also are presented. In the range $-0.1 < C < 0.1$, Tani's formula is in good agreement with the present result. The deviation of Murphy's result from the present one may be attributed to his questionable use of series expansion for the determination of C_f. Yen and Toba's result seems to be erroneous. It may be suspected that they started their numerical calculation from a point too near to the surface, where their equation has a singularity, and that they used integration steps that are too large for treating such an equation.

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Some Physical Interpretations of Magnetohydrodynamic Duct Flows

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This note presents some physical interpretations of magnetohydrodynamic duct flows with various boundary conditions viewed in the light of the effects of conducting walls on the pattern of electric current, taking examples from published results on rectangular ducts.[1–4] The current patterns are illustrated in Fig. 1 for rectangular ducts having various combinations of conducting and nonconducting walls, a uniform magnetic field being applied in the horizontal direction.

There is an essential difference between the roles played by horizontal and vertical conducting walls. A horizontal conducting wall serves only as an electrode (cases A[1] and B[2]) or as a short cut for the current (case D[2]). Therefore the mechanism of flow resistance in case D remains essentially the same as in a duct of nonconducting walls (case E[3]). On the contrary, a vertical conducting wall acts essentially to pass the current in the vertical direction outside of the fluid, thus resulting in a net current in the fluid which makes a primary contribution to flow resistance at large Hartmann number $M = B_0a(\sigma/\eta)^{1/2}$.

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